Two-Way Factorial ANOVA

PSYC 300B - Lecture 3
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Two-Way ANOVA

- An ANOVA involving all combinations of the levels of two factors (IVs)
- Factors can be between-subjects, within-subjects, or a mix of the two
- Factorial designs have several advantages over one-way designs: in particular they allow us to look at the interaction between the independent variables

Two-Way ANOVA Assumptions

- Data within each condition are normally distributed
- Variance across conditions is homogeneous

2 × 3 between-groups ANOVA with the factors Gender (male vs. female) and Alcohol (0 vs. 2 vs. 4) on on the DV Attractiveness of date

Alcohol	None		2 Pints		4 Pints	
Gender	Female	Male	Female	Male	Female	Male
	65	50	70	45	55	30
	70	55	65	60	65	30
	60	80	60	85	70	30
	60	65	70	65	55	55
	60	70	65	70	55	35
	55	75	60	70	60	20
	60	75	60	80	50	45
	55	65	50	60	50	40

Hypothesis Testing

- Two-way ANOVA simultaneously tests 3 hypotheses:
- ***Main effect of Factor A (Gender):*** Does a difference exist between the gender means?
- ***Main effect of Factor B (Alcohol):*** Does a difference exist between the alcohol means?
- ***A × B Interaction (Gender × Alcohol) :*** Does a difference exist between any of the unique combinations of means for gender and alcohol?

Main Effects and Interactions

- Main effects describe significant differences between the means within a factor
- An interaction exists when the effects of one factor on the dependent variable, depend on the levels of the other factor
- When the interaction is significant main effects are typically considered unimportant (Howell, 2014)
- Generally, you do not interpret the main effects when there is a significant interaction (Field, 2013, p. 528)

Main Effect: Gender

A_1: Female

65	70	55
70	65	65
60	60	70
60	70	55
60	65	55
55	60	60
60	60	50
55	50	50

Mean Female = 60.21

A_2: Male

50	45	30
55	60	30
80	85	30
65	65	55
70	70	35
75	70	20
75	80	45
65	60	40

Mean Male = 56.46

Main Effect: Alcohol

B_1: None

65	50
70	55
60	80
60	65
60	70
55	75
60	75
55	65

Mean None = 63.75

B_2: 2 Pints

70	45
65	60
60	85
70	65
65	70
60	70
60	80
50	60

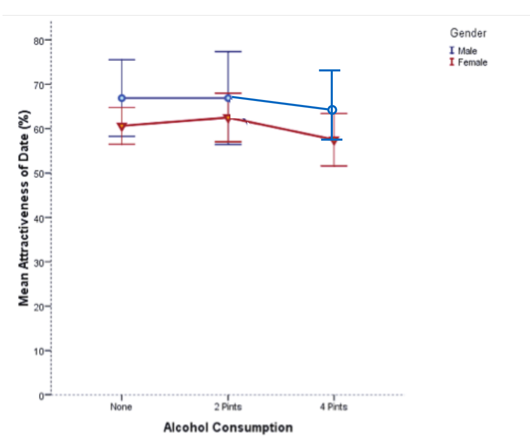
Mean 2 Pints = 64.69

B_3: 4 Pints

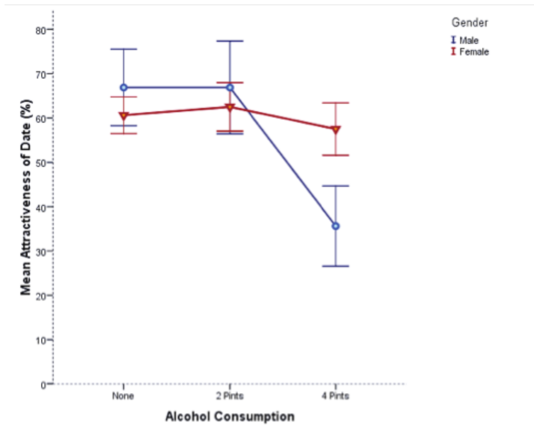
55	30
65	30
70	30
55	55
55	35
60	20
50	45
50	40

Mean 4 Pints = 46.56

No Gender × Alcohol Interaction



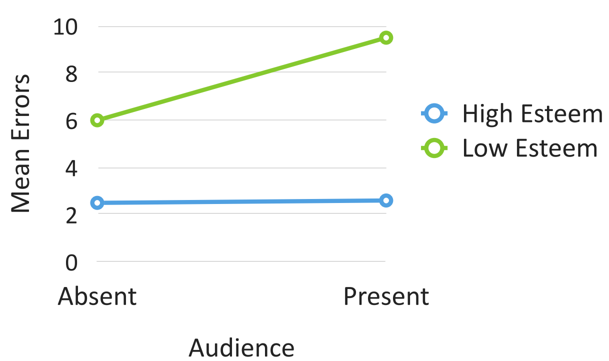
Interaction: Gender × Alcohol



2 × 2 between-groups ANOVA with the factors Self-esteem (high vs. low) and Audience (present vs. absent) on the dependent variable Speaking Errors

		Factor *B*: Audience Condition	
		No Audience	Audience
Factor *A*: Self-Esteem	Low	Scores for a group of participants who are classified as low self-esteem and are tested with no audience.	Scores for a group of participants who are classified as low self-esteem and are tested with an audience.
	High	Scores for a group of participants who are classified as high self-esteem and are tested with no audience.	Scores for a group of participants who are classified as high self-esteem and are tested with an audience.

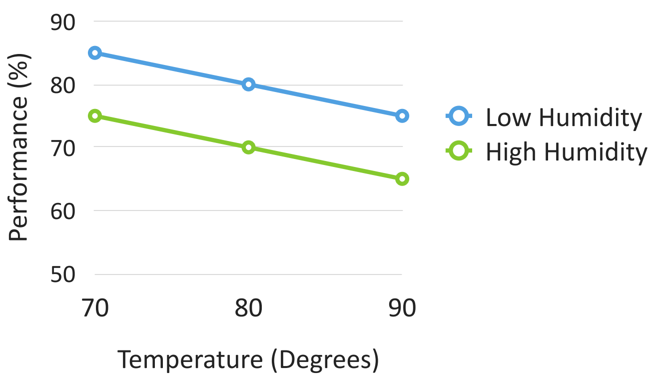
Audience × Esteem interaction: the effect of audience on errors ***depends*** *on the level of self-esteem*



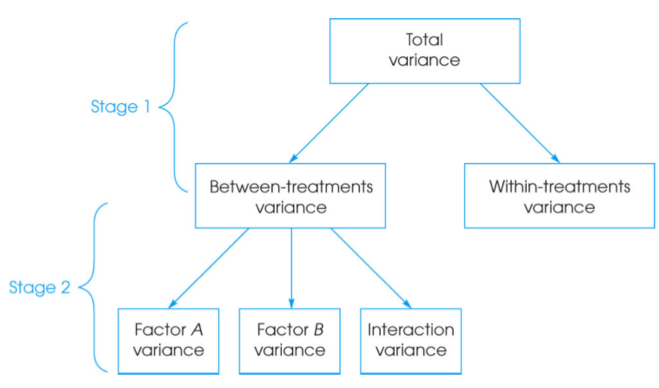
2 × 3 between-groups ANOVA with the factors Humidity (high vs. low) and Temperature (70 vs. 80 vs. 90) on the dependent variable Test Performance

		Factor *B*: Temperature		
		70° Room	80° Room	90° Room
Factor A: Humidity	Low Humidity	Scores for $n = 15$ participants tested in a 70° room with low humidity	Scores for $n = 15$ participants tested in an 80° room with low humidity	Scores for $n = 15$ participants tested in a 90° room with low humidity
	High Humidity	Scores for $n = 15$ participants tested in a 70° room with high humidity	Scores for $n = 15$ participants tested in an 80° room with high humidity	Scores for $n = 15$ participants tested in a 90° room with high humidity

*No interaction: the effect of temperature on performance **does not depend** on humidity*



Partitioning Variance in a Two-Way ANOVA



Calculating the Sum of Squares

$$SS_{TOTAL} = \sum (X - M_{GRAND})^2$$

$$SS_A = n(k_B) \sum (M_{A_i} - M_{GRAND})^2$$

$$SS_B = n(k_A) \sum (M_{B_i} - M_{GRAND})^2$$

$$SS_{CELLS} = n \sum (M_{CELL} - M_{GRAND})^2$$

$$SS_{A \times B} = SS_{CELLS} - SS_A - SS_B$$

$$SS_{WITHIN} = SS_{TOTAL} - SS_{CELLS}$$

Calculating the Degrees of Freedom

$$df_{TOTAL} = N - 1$$

$$df_A = k_A - 1$$

$$df_B = k_B - 1$$

$$df_{A \times B} = df_A \times df_B$$

$$df_{WITHIN} = df_{TOTAL} - df_A - df_B - df_{A \times B}$$

Calculating the Mean Squares

$$MS_A = \frac{SS_A}{df_A}$$

$$MS_B = \frac{SS_B}{df_B}$$

$$MS_{A \times B} = \frac{SS_{A \times B}}{df_{A \times B}}$$

$$MS_{WITHIN} = \frac{SS_{WITHIN}}{df_{WITHIN}}$$

Calculating the *F*-Ratio

$$F_A = \frac{MS_A}{MS_{WITHIN}}$$

$$F_B = \frac{MS_B}{MS_{WITHIN}}$$

$$F_{A\times B} = \frac{MS_{A\times B}}{MS_{WITHIN}}$$

Calculating the *F*-Critical Values

$$F_{CRITICAL\ A} = (df_A, df_{WITHIN})$$

$$F_{CRITICAL\ B} = (df_B, df_{WITHIN})$$

$$F_{CRITICAL\ A\times B} = (df_{A\times B}, df_{WITHIN})$$

Effect Size Measures

$$\eta^2{}_A = \frac{SS_A}{SS_A + SS_{WITHIN}}$$

$$\eta^2{}_B = \frac{SS_B}{SS_B + SS_{WITHIN}}$$

$$\eta^2{}_{AxB} = \frac{SS_{AxB}}{SS_{AxB} + SS_{WITHIN}}$$

Effect Size Measures

$$\omega^2{}_A = \frac{SS_A - (k_A - 1)MS_{WITHIN}}{SS_{TOTAL} + MS_{WITHIN}}$$

$$\omega^2{}_B = \frac{SS_B - (k_B - 1)MS_{WITHIN}}{SS_{TOTAL} + MS_{WITHIN}}$$

$$\omega^2{}_{A\times B} = \frac{SS_{A\times B} - (k_A - 1)(k_B - 1)MS_{WITHIN}}{SS_{TOTAL} + MS_{WITHIN}}$$

$$d = \frac{M_1 - M_2}{\sqrt{MS_{WITHIN}}}$$

Post-Hoc Tests

$$Tukey's\ HSD = q\sqrt{\frac{MS_{WITHIN}}{n}}$$

$$Fisher's\ LSD\ test: t = \frac{M_1 - M_2}{\sqrt{MS_{WITHIN}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

A 2 × 5 between-groups experimental design with n = 10 participants in each condition (N=100)

	Count	Rhyme	Adjective	Imagery	Intention	*M*
Old	7.0	6.9	11.0	13.4	12.0	10.06
Young	6.5	7.6	14.8	17.6	19.3	13.16
M	6.75	7.25	12.9	15.5	15.65	**11.61**

Cell values reflect mean recall memory for that condition

Is there an effect of depth of processing on recall memory?

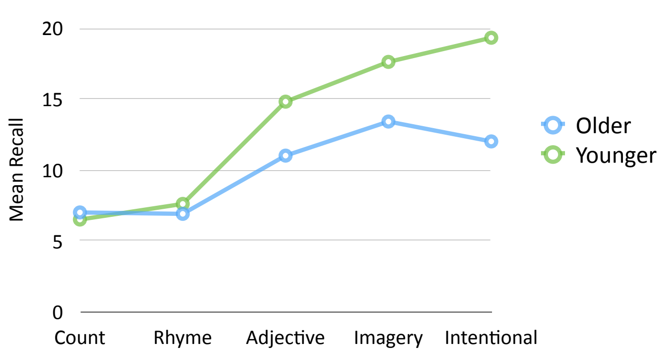
Is there an effect of age on recall memory?

Is there an interaction between depth of processing and age on recall memory?

Source	*SS*	*df*	*MS*	*F*
Between/Cells	1945.49			
- Age	240.25	1	240.25	29.94*
- Encoding	1514.94	4	378.74	47.19*
- Age × Encoding	190.30	4	47.58	5.93*
Within	722.30	90	8.03	
Total	2667.79	99		

*$p < .05$

*The effect of encoding condition on memory **depends** on age*



Simple Effects Analysis

- Used to analyze a significant interaction
- The analysis looks at the effect of one factor at each level of the other factor
- If possible perform the analysis in such a way that you are just comparing the means of two conditions

Simple Effects Analysis

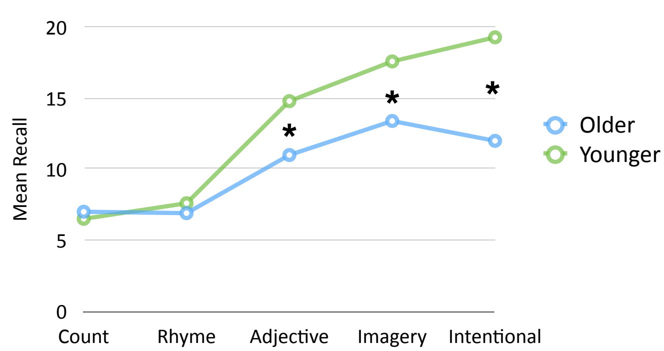
$$SS_{SIMPLE\ EFFECTS} = n\sum(M_{CELL} - M_{ROW\ OR\ COLUMN})^2$$

$$MS_{SIMPLE\ EFFECTS} = \frac{SS_{SIMPLE\ EFFECTS}}{df} = \frac{SS_{SIMPLE\ EFFECTS}}{\#\ of\ conditions - 1}$$

$$F_{SIMPLE\ EFFECTS} = \frac{MS_{SIMPLE\ EFFECTS}}{MS_{WITHIN}}$$

$$F_{CRITICAL} = (df_{SIMPLE\ EFFECTS}, df_{WITHIN})$$

Older adults perform significantly poorer than younger adults in conditions that require deeper processing



*p < .05

- An company believes that showing TV commercials at louder levels than the TV program will draw viewers' attention and make the commercial more persuasive
- To test the theory, 9 males and 9 females are assigned to one of three volume groups: soft, medium, and loud
- After viewing the commercials participants provide persuasiveness ratings about the claims made in the adverts
- Conduct a hypothesis test (α = .05) to determine if there is:
  - An effect of volume on persuasiveness ratings
  - An effect of gender on persuasiveness ratings
  - An interaction between volume and gender on persuasiveness ratings

Between-subjects design with n = 3 (N = 18) participants in each condition

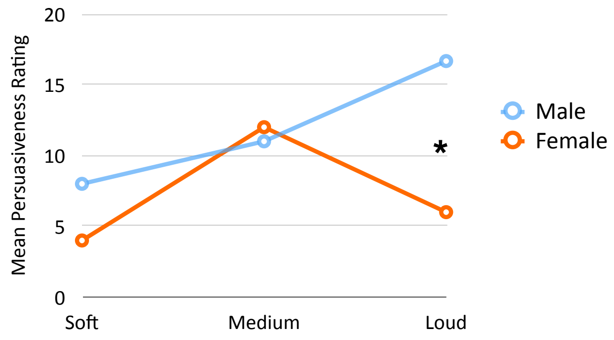
	Soft	Medium	Loud	*M*
Male	8.0	11.0	16.67	11.89
Female	4.0	12.0	6.0	7.33
M	6.0	11.5	11.33	**9.61**

Cell values reflect mean persuasiveness ratings

Source	*SS*	*df*	*MS*	*F*
Between/Cells				
- Gender	93.39	1	93.39	11.36*
- Volume	117.45	2	58.73	7.14*
- Gender × Volume	102.77	2	51.39	6.25*
Within	98.67	12	8.22	
Total	412.28	17		

*p < .05

Males were significantly more persuaded than females at the loud volume level



*$p < .05$

- A drug company has developed a diet pill to facilitate weight-loss by suppressing appetite
- The company tests the efficacy of the drug by randomly selecting 15 males and 15 females and assigning them to a placebo, low-dose, or high-dose condition
- Eating behaviour is then measured as the change in food consumption over a one-week period
- Conduct a hypothesis test ($\alpha = .05$) to determine if there is:
  - An effect of dosage on food consumption
  - An effect of gender on food consumption
  - An interaction between dosage and gender on food consumption

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Between-subjects design with n = 5 (N = 30) participants in each condition

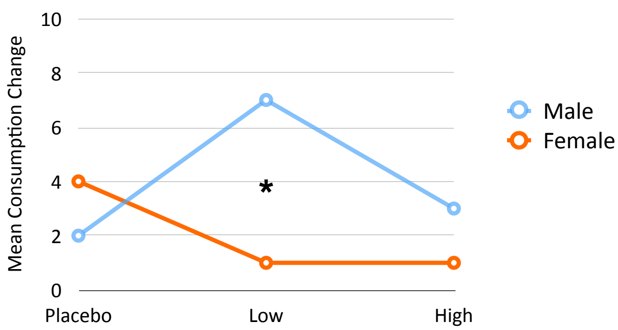
	Placebo	Low	High	*M*
Male	2.0	7.0	3.0	4.0
Female	4.0	1.0	1.0	2.0
M	3.0	4.0	2.0	**3.0**

Cell values reflect mean reduction in food consumption (lbs.)

Source	*SS*	*df*	*MS*	*F*
Between/Cells				
- Gender	30.0	1	30.0	6.0*
- Dose	20.0	2	10.0	2.0
- Gender × Dose	80.0	2	40.0	8.0*
Within	120.0	24	5.0	
Total	250.0	29		

*$p < .05$

Males in the low dose condition had a significantly larger reduction in food consumption than females



*$p < .05$

Between-subjects design with n = 8 (N = 48) participants in each condition

	Placebo	Low	High	*M*
Attractive	6.38	6.50	6.13	6.33
Unattractive	3.50	4.87	6.63	5.00
M	4.94	5.69	6.37	**5.67**

Cell values reflect mean attractiveness ratings

Source	*SS*	*df*	*MS*	*F*
Between/Cells				
- Facetype	21.33	1	21.33	15.58*
- Alcohol	16.54	2	8.27	6.04*
- Facetype × Alcohol	23.29	2	11.65	8.50*
Within	57.5	42	1.37	
Total	118.67	47		

*$p < .05$

The effect of alcohol on attractiveness ratings depends on facetype